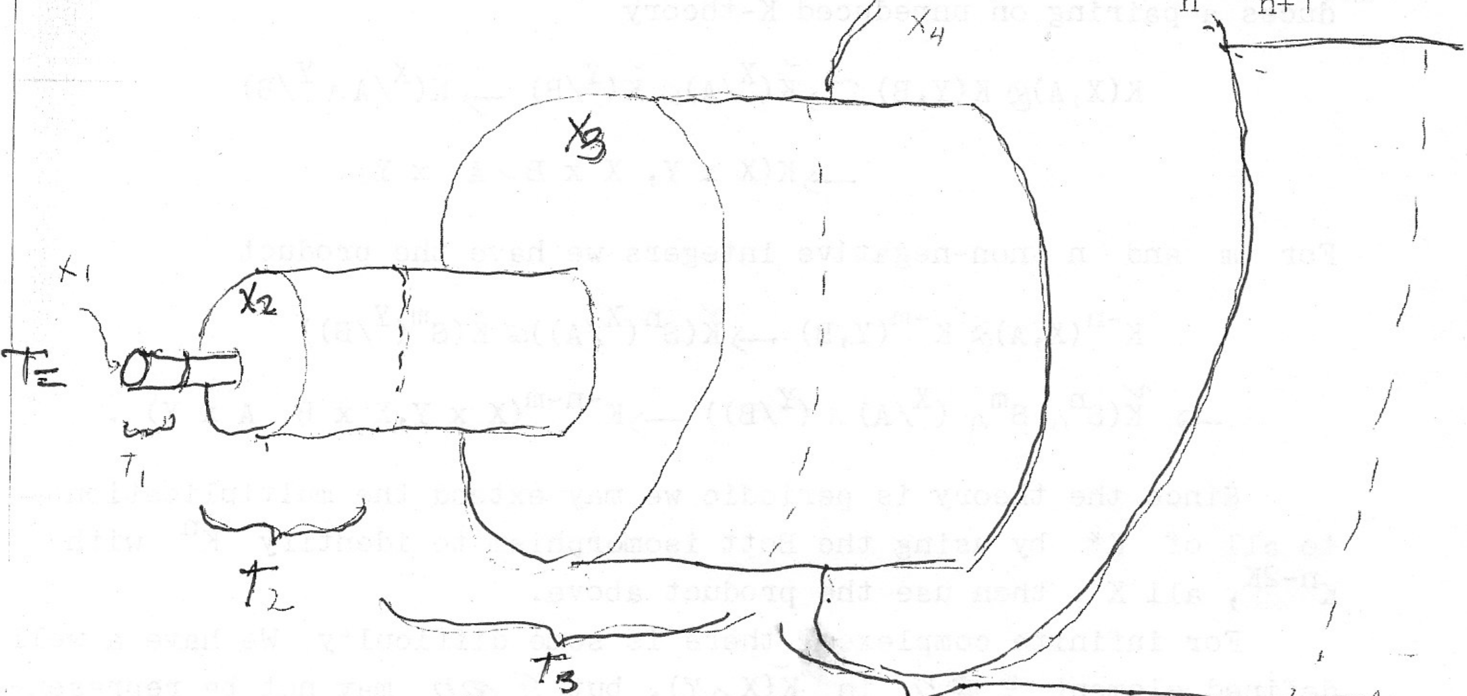
6.2 Theorem. $\iota : \tilde{h}^*(X) \to \varprojlim \tilde{h}^*(X_n)$ is onto. If $\tilde{h}^*(X_{n+1}) \xrightarrow{in^*} \tilde{h}^*(X_n)$ is onto for all n, then ι is an isomorphism.

Remark. In general the kernal of ι is $\varprojlim^1 \tilde{h}^{*-1}(X_n)$, the first derived functor of $\varprojlim$. This is a theorem of Milnor [12].

Proof. Let T be the mapping telescope of $\ldots X_n \subset X_{n+1} \ldots$



The mapping telescope

Let T_i be as in the figure. Define $T^0 = \cup T_{odd}$ $T^1 = \cup T_{even}$. Then $T_i \sim X_i$; $T^0 \cap T^1 = \cup X_i$ and under the inclusion $\beta_\varepsilon : T^0 \cap T^1 \to T^\varepsilon$ ($\varepsilon = 0$ or 1) $X_i \Rightarrow X_i \sim T_i$ if $T_i \in T^\varepsilon$. $X_i \hookrightarrow X_{i+1} \sim T_{i+1}$ if $T_i \notin T^\varepsilon$. ~~Hence we have shown:~~

$\underline{\underline{6.3}}$. Consider the map $\pi \tilde{h}^*(X_n) = \tilde{h}^*(T^0) \oplus \tilde{h}^*(T^1)$
$\longrightarrow \pi h^*(X_n) = \tilde{h}^*(T^0 \cap T^1)$ given
by $\beta_0 - \beta_1$. Then $(\alpha_1, \alpha_2, \ldots) \longmapsto (i_1{}^*\alpha_2 - \alpha_1, \alpha_2 - i_2{}^*\alpha_3, i_3{}^*\alpha_4 - \alpha_3, \ldots$
Now consider the m.v. sequence for (T, T^0, T^1)

$$\longrightarrow \tilde{h}^{*-1}(T_0 \cap T_1) \xrightarrow{\delta} \tilde{h}^*(X) \xrightarrow{1} \tilde{h}^*(T^0) \oplus \tilde{h}^*(T^1) \xrightarrow{\beta_0 - \beta_1} \tilde{h}^*(T^0 \cap T^1)$$

$6.3 \Longrightarrow (\beta_0 - \beta_1)(\varprojlim \tilde{h}^*(X_n))$ is zero hence exactness implies the first part of 6.2.

Now suppose $i_n{}^*$ is onto. for all n. We prove that $\beta_0 - \beta_1$ is onto $(\Longrightarrow \delta = 0)$ proving the theorem.

Let $\lambda = (\lambda_1, \lambda_2, \ldots) \in \pi \tilde{h}^*(X_n) = \tilde{h}^*(T^0 \cap T^1)$. We construct $(\alpha_1, \alpha_2, \ldots) \in \tilde{h}^*(T^0) \oplus \tilde{h}^*(T^1)$ which maps to λ.

Let $\alpha_1 = 0$, α_2 is any element such that $i_1{}^*(\alpha_2) = \lambda_1$. α_3 is defined to be any element satisfying $i_2{}^*(\alpha_3) = \alpha_2 - \lambda_3$, etc.
Q.E.D.