

## Topics in homotopy theory

1. Stable homotopy categories. The definition of Spectra and omega spectra. Perhaps the Boardman Category.
2. Definition of homology cohomology and homotopy of spaces and spectra defined in terms of a spectrum.
3. Examples: MU (complex cobordism) Ordinary homology, Morava K-theory, Wilson Johnson homology theory
4. Formal group laws with application of construction of the BP spectrum from MU.
5. The Adams Novikov spectral sequence. Application of  $K(n)$  (Morava K-theory) to the Adams-Novikov Spectral sequence and the stable homotopy groups of spheres.
6. Localization and completion with respect to a homology theory.
7. Putting it all together to construct the chromatic spectral sequence with applications computation of the  $E_2$  term of the Adams-Novikov Spectral sequence.
8. If time the change of rings theorem and the Morava Stabilizer algebra.

More details will follow.

Some references

For some of the constructions used so far, e.g. the smash product, based loop space. With all the point set conditions that make everything work:

Algebraic Topology by Maunder

<http://www.maths.ed.ac.uk/aar/papers/maunder.pdf>

A book by Peter May on Algebraic Topology.

<http://www.maths.ed.ac.uk/aar/papers/maybook.pdf>

Reference for Category Theory

<http://katmat.math.uni-bremen.de/acc/acc.pdf>

And notes by Tyler Bryson

<https://www.dropbox.com/s/94d5ka5c5idbg2b/Category>

Milnor's exact sequence

[math.hunter.cuny.edu/mbenders/Milnor.pdf](http://math.hunter.cuny.edu/mbenders/Milnor.pdf)

For the Steenrod algebra (for the prime 2)

Cohomology Operations and Applications in Homotopy Theory

by Mosher and Tangora

The reference for the proof that spectra define a homology theory

G Whitehead: Generalized homology theories, Trans AMS 102 (1962) pages 227-283

<http://www.ams.org/journals/tran/1962-102-02/S0002-9947-1962-0137117-6/S0002-9947-1962-0137117-6.pdf>

Doug Ravenel's book:  
Complex Cobordism and Stable Homotopy Groups of Spheres

Stong's book:  
Notes on Cobordism Theory

Tables of BP structure maps.

[math.hunter.cuny.edu/mbenders/Tables.pdf](http://math.hunter.cuny.edu/mbenders/Tables.pdf)

I will take a rather naive approach to spectra that works just fine for stable homotopy, but is not structured enough for certain constructions in the stable category, e.g. homotopy fixed points. One can easily spend a year introducing the complete theory of spectra.

Here are some references if you want some of the omitted details:

Lectures on Spectra:

A.D. Elmendorf, I.Kriz, M.A. Mandell, and J. P.May

<http://www.math.uchicago.edu/~may/BOOKS/EKMM.pdf>

Pairings of ring spectra may be found in

M. A. MANDELL, J.P. MAY, S. SCHWEDE, AND  
B. SHIPLEY

<http://www.math.uiuc.edu/K-theory/0319/mmss1nov14.pdf>