and the coproduct $\psi: BP_*(BP) \to BP_*(BP)$ given by

$$\sum_{i+j=k} m_i \psi(t_j)^{p^i} = \sum_{h+i+j=k} m_h t_i^{p^h} \otimes t_j^{p^{h+i}} .$$

Actually $\eta_R(v_i)$, $c(t_i)$ and $\psi(t_i)$ are computed up through the first case which is not easy $(i \leq 3)$. It would not be difficult to compute more, but the number of terms involved make it seem unlikely to be useful. However modification to the programs to select specific terms would not be difficult.

<u>PART 1: exp(X) mod X^{17} for $p = 2$</u>

$$\begin{aligned}
\exp(X) = {} & X - m_1X^2 + 2m_1^2X^3 - (m_2 + 5m_1^3)X^4 + (6m_1m_2 + 14m_1^4)X^5 - (28m_1^2m_2 - 42m_1^5)X^6 \\
& + (120m_1^3m_2 + 132m_1^6 + 4m_2^2)X^7 - (m_3 + 45m_1m_2^2 + 495m_1^4m_2 + 429m_1^7)X^8 \\
& + (10m_1m_3 + 330m_1^2m_2^2 + 2002m_1^5m_2 + 1430m_1^8)X^9 \\
& - (66m_1^2m_3 + 2002m_1^3m_2^2 + 8008m_1^6m_2 + 4862m_1^9 + 22m_2^3)X^{10} \\
& + (364m_1m_2^3 + 12m_2m_3 + 364m_1^3m_3 + 10920m_1^4m_2^2 + 31824m_1^7m_2 + 16796m_1^{10})X^{11} \\
& - (182m_1m_2m_3 + 3640m_1^2m_2^3 + 1820m_1^4m_3 + 55692m_1^5m_2^2 + 125970m_1^8m_2 + 58786m_1^{11})X^{12} \\
& + (1680m_1^2m_2m_3 + 28560m_1^3m_2^3 + 8568m_1^5m_3 + 271320m_1^6m_2^2 + 497420m_1^9m_2 \\
& \qquad + 208012m_1^{12} + 140m_2^4)X^{13} \\
& - (3060m_1m_2^4 + 12240m_1^3m_2m_3 + 193800m_1^4m_2^3 + 38760m_1^6m_3 + 1279080m_1^7m_2^2 \\
& \qquad + 1961256m_1^{10}m_2 + 120m_2^2m_3 + 742900m_1^{13})X^{14} \\
& + (2448m_1m_2^2m_3 + 38760m_1^2m_2^4 + 77520m_1^4m_2m_3 + 1193808m_1^5m_2^3 + 170544m_1^7m_3 \\
& \qquad + 5883768m_1^8m_2^2 + 7726160m_1^{11}m_2 + 2674440m_1^{14} + 8m_3^2)X^{15} \\
& - (m_4 + 153m_1m_3^2 + 29070m_1^2m_2^2m_3 + 373065m_1^3m_2^4 + 447678m_1^5m_2m_3 + 6864396m_1^6m_2^3 \\
& \qquad + 735471m_1^8m_3 + 26558675m_1^9m_2^2 + 30421775m_1^{12}m_2 + 9694845m_1^{15} \\
& \qquad + 969m_2^5)X^{16}
\end{aligned}$$

<u>exp(X) mod X^{28} for $p = 3$</u>

$$\begin{aligned}
\exp(X) = {} & X - m_1X^3 + 3m_1^2X^5 - 12m_1^3X^7 - (m_2 - 55m_1^4)X^9 + (12m_1m_2 - 273m_1^5)X^{11} \\
& - (105m_1^2m_2 - 1428m_1^6)X^{13} + (816m_1^3m_2 - 7752m_1^7)X^{15} \\
& - (5985m_1^4m_2 - 43263m_1^8 - 9m_2^2)X^{17} - (210m_1m_2^2 - 42504m_1^5m_2 + 246675m_1^9)X^{19} \\
& + (3036m_1^2m_2^2 - 296010m_1^6m_2 + 1430715m_1^{10})X^{21}
\end{aligned}$$

$$- (35100m_1^3m_2^2 - 2035800m_1^7m_2 + 8414640m_1^{11})X^{23}$$
$$+ (356265m_1^4m_2^2 - 13884156m_1^8m_2 + 50067108m_1^{12} - 117m_2^3)X^{25}$$
$$- (m_3 - 4060m_1m_2^3 + 3322704m_1^5m_2^2 - 94143280m_1^9m_2 + 300830572m_1^{13})X^{27}$$

[2](X) for $p = 2$ mod X^{17} in terms of the m_i

$$[2](X) = 2X - 2m_1X^2 + 8m_1^2X^3 - (14m_2 + 36m_1^3)X^4 + (120m_1m_2 + 176m_1^4)X^5$$
$$- (888m_1^2m_2 + 912m_1^5)X^6 + (6240m_1^3m_2 + 4928m_1^6 + 448m_2^2)X^7$$
$$- (254m_3 + 7172m_1m_2^2 + 42848m_1^4m_2 + 27472m_1^7)X^8$$
$$+ (3064m_1m_3 + 80496m_1^2m_2^2 + 290816m_1^5m_2 + 156864m_1^8)X^9$$
$$- (28632m_1^2m_3 + 775024m_1^3m_2^2 + 1961472m_1^6m_2 + 912832m_1^9 + 19040m_2^3)X^{10}$$
$$+ (451264m_1m_2^3 + 22464m_2m_3 + 239392m_1^3m_3 + 6850240m_1^4m_2^2 + 13183744m_1^7m_2 + 5394176m_1^{10})X^{11}$$
$$- (442632m_1m_2m_3 + 6814928m_1^2m_2^3 + 1882320m_1^4m_3 + 57356064m_1^5m_2^2 + 88443968m_1^8m_2 + 32282240m_1^{11})X^{12}$$
$$+ (5811360m_1^2m_2m_3 + 83369280m_1^3m_2^3 + 14245952m_1^5m_3 + 462765184m_1^6m_2^2 + 592746752m_1^9m_2 + 195264000m_1^{12} + 932288m_2^4)X^{13}$$
$$- (29319872m_1m_2^4 + 63378208m_1^3m_2m_3 + 900627520m_1^4m_2^3 + 105098688m_1^6m_3 + 3635159424m_1^7m_2^2 + 3970848000m_1^{10}m_2 + 1699520m_2^2m_3 + 1191825920m_1^{13})X^{14}$$
$$+ (46522752m_1m_2^2m_3 + 556855040m_1^2m_2^4 + 620457600m_1^4m_2m_3 + 8960342272m_1^5m_2^3 + 7331457024m_1^{14} + 761561344m_1^7m_3 + 27987144192m_1^8m_2^2 + 26598675456m_1^{11}m_2 + 260096m_3^2)X^{15}$$
$$- (65534m_4 + 6063108m_1m_3^2 + 789099408m_1^2m_2^2m_3 + 8262562000m_1^3m_2^4 + 5659676160m_1^5m_2m_3 + 84080594304m_1^6m_2^3 + 5446592576m_1^8m_3 + 212145331840m_1^9m_2^2 + 178193961216m_1^{12}m_2 + 45406194944m_1^{15} + 49590800m_2^5)X^{16}$$

[2](X) for $p = 2$ mod X^{17} in terms of the v_i

$$[2](X) = 2X - v_1X^2 + 2v_1^2X^3 - (7v_2 + 8v_1^3)X^4 + (30v_1v_2 + 26v_1^4)X^5$$
$$- (111v_1^2v_2 + 84v_1^5)X^6 + (502v_1^3v_2 + 300v_1^6 + 112v_2^2)X^7$$

$$- (127v_3 + 960v_1v_2^2 + 2299v_1^4v_2 + 1140v_1^7)X^8$$
$$+ (766v_1v_3 + 5414v_1^2v_2^2 + 9958v_1^5v_2 + 4334v_1^8)X^9$$
$$- (3579v_1^2v_3 + 29579v_1^3v_2^2 + 43118v_1^6v_2 + 16692v_1^9 + 2380v_2^3)X^{10}$$
$$+ (31012v_1v_2^3 + 5616v_2v_3 + 17770v_1^3v_3 + 161034v_1^4v_2^2 + 189976v_1^7v_2 + 65744v_1^{10})X^{11}$$
$$- (55329v_1v_2v_3 + 240631v_1^2v_2^3 + 86487v_1^4v_3 + 838452v_1^5v_2^2 + 837637v_1^8v_2 + 262400v_1^{11})X^{12}$$
$$+ (363210v_1^2v_2v_3 + 1600786v_1^3v_2^3 + 404198v_1^5v_3 + 4232750v_1^6v_2^2 + 368550v_1^9v_2 + 1056540v_1^{12} + 58268v_2^4)X^{13}$$
$$- (1022466v_1v_2^4 + 2193009v_1^3v_2v_3 + 10071369v_1^4v_2^3 + 1864478v_1^6v_3 + 21110372v_1^7v_2^2 + 1625450v_1^{10}v_2 + 212440v_2^2v_3 + 4292816v_1^{13})X^{14}$$
$$+ (2972696v_1v_2^2v_3 + 10170952v_1^2v_2^4 + 12667346v_1^4v_2v_3 + 60190566v_1^5v_2^3 + 8581604v_1^7v_3 + 104219628v_1^8v_2^2 + 71867828v_1^{11}v_2 + 17587492v_1^{14} + 65024v_3^2)X^{15}$$
$$- (32767v_4 + 774272v_1v_3^2 + 25417245v_1^2v_2^2v_3 + 80952889v_1^3v_2^4 + 69633465v_1^5v_2v_3 + 344343134v_1^6v_2^3 + 39306153v_1^8v_3 + 509125669v_1^9v_2^2 + 318135602v_1^{12}v_2 + 72547972v_1^{15} + 1566096v_2^5)X^{16}$$

$[3](X) \bmod X^{28}$ for $p = 3$ in terms of the m_i

$$[3](X) = 3X - 24m_1X^3 + 648m_1^2X^5 - 22680m_1^3X^7 - (19680m_2 - 906120m_1^4)X^9$$
$$+ (1948536m_1m_2 - 39161880m_1^5)X^{11} - (144725616m_1^2m_2 - 1782778248m_1^6)X^{13}$$
$$+ (9647551656m_1^3m_2 - 84205559448m_1^7)X^{15}$$
$$- (609973825536m_1^4m_2 - 4088238304392m_1^8 - 11620843m_2^2)X^{17}$$
$$- (224294496984m_1m_2^2 - 37443174594264m_1^5m_2 + 202766127578136m_1^9)X^{19}$$
$$+ (27276329260728m_1^2m_2^2 - 2257636316956560m_1^6m_2 + 10229293584254088m_1^{10})X^{21}$$
$$- (2678311213085008m_1^3m_2^2 - 134569318031340552m_1^7m_2 + 523275105375281304m_1^{11})X^{23}$$
$$+ (232282027220756400m_1^4m_2^2 - 7959882821409557280m_1^8m_2 + 270790967406439694 16m_1^{12} - 991130095684800m_2^3)X^{25}$$
$$- (762559748498 4m_3 - 28477285430249016m_1m_2^3$$

$$+ 18585098326160051112m_1^5m_2^2 - 468355228225966841400m_1^9m_2$$
$$+ 1415075117768856781848m_1^{13})X^{27}$$

[3](X) for p = 3 mod X^{28} in terms of the v_i

$$[3](X) = 3X - 8v_1X^3 + 72v_1^2X^5 - 840v_1^3X^7 - (6560v_2 - 9000v_1^4)X^9$$
$$+ (216504v_1v_2 - 88992v_1^5)X^{11} - (5360208v_1^2v_2 - 658776v_1^6)X^{13}$$
$$+ (119105576v_1^3v_2 + 1199088v_1^7)X^{15} - (2424100032v_1^4v_2 + 199267992v_1^8$$
$$- 129120480v_2^2)X^{17}$$
$$- (8307202392v_1v_2^2 - 45824243688v_1^5v_2 - 5896183992v_1^9)X^{19}$$
$$+ (336744805688v_1^2v_2^2 - 807801733088v_1^6v_2 - 133449348816v_1^{10})X^{21}$$
$$- (11021856839856v_1^3v_2^2 - 13162584394728v_1^7v_2 - 2658275605728v_1^{11})X^{23}$$
$$+ (314960186505360v_1^4v_2^2 - 193206850384 0v_1^8v_2 - 48579725371464v_1^{12}$$
$$- 3670852206240v_2^3)X^{25}$$
$$- (2541865828328v_3 - 350724136455360v_1v_2^3 + 8146415921144640v_1^5v_2^2$$
$$- 2382655204483352v_1^9v_2 - 824825727922536v_1^{13})X^{27}$$

The coefficients a_{ij} of F(X,Y) for p = 2 w_i = Araki

$a_{11} = - 2m_1 = - v_1$ $= w_i$

$a_{12} = 4m_1^2 = v_1^2$ $= w_i^2$

$a_{13} = - 4m_2 - 8m_1^3 = - 2v_2 - 2v_1^3$

$a_{14} = 16m_2m_1 + 16m_1^4 = 4v_2v_1 + 3v_1^4$

$a_{15} = - 48m_1^2m_2 - 32m_1^5 = - 6v_1^2v_2 - 4v_1^5$

$a_{16} = 128m_1^3m_2 + 64m_1^6 + 16m_2^2 = 4v_2^2 + 12v_1^3v_2 + 6v_1^2$

$a_{17} = - 8m_3 - 96m_1m_2^2 - 320m_2m_1^4 - 128m_1^7 = - 4v_3 - 14v_2^2v_1 - 24v_2v_1^4 - 10v_1^7$

$a_{18} = 32m_3m_1 + 384m_2^2m_1^2 + 768m_2m_1^5 + 256m_1^8 = 8v_3v_1 + 28v_2^2v_1^2 + 40v_1^5v_2 + 15v_1^8$

$a_{22} = - 6m_2 - 20m_1^3 = - 3v_2 - 4v_1^3$

$a_{23} = 44m_2m_1 + 72m_1^4 = 11v_2v_1 + 10v_1^4$

$a_{24} = - 224m_2m_1^2 - 224m_1^5 = - 28v_2v_1^2 - 21v_1^5$

$a_{25} = 72m_2^2 + 912m_2m_1^3 + 640m_1^6 = 18v_2^2 + 75v_1^3v_2 + 43v_1^6$

$$a_{26} = -28m_3 - 656m_2^2m_1 - 3232m_2m_1^4 - 1728m_1^7 = -14v_3 - 89v_2^2v_1 - 190v_2v_1^4 - 88v_1^7$$

$$a_{27} = 184m_3m_1 + 3744m_2^2m_1^2 + 10432m_2m_1^5 + 4480m_1^8 = 46v_3v_1 + 257v_2^2v_1^2 + 420v_2v_1^5 + 169v_1^8$$

$$a_{33} = -344m_2m_1^2 - 400m_1^5 = -43v_2v_1^2 - 34v_1^5$$

$$a_{34} = 136m_2^2 + 2080m_2m_1^3 + 1760m_1^6 = 34v_2^2 + 164v_2v_1^3 + 101v_1^6$$

$$a_{35} = -56m_3 - 1696m_2^2m_1 - 10400m_2m_1^4 - 6720m_1^7 = -28v_3 - 226v_2^2v_1 - 551v_2v_1^4 - 275v_1^7$$

$$a_{36} = 504m_3m_1 + 13056m_2^2m_1^2 + 45248m_2m_1^5 + 23296m_1^8$$
$$= 126v_3v_1 + 879v_2^2v_1^2 + 1586v_2v_1^5 + 680v_1^8$$

$$a_{44} = -70m_3 - 2276m_2^2m_1 - 14944m_1^4m_2 - 10320m_1^7 = -35v_3 - 302v_2^2v_1 - 769v_2v_1^4 - 394v_1^7$$

$$a_{45} = 812m_3m_1 + 23064m_2^2m_1^2 + 88960m_1^5m_2 + 50400m_1^8$$
$$= 203v_3v_1 + 1543v_2^2v_1^2 + 2933v_2v_1^5 + 1303v_1^8$$

The coefficients a_{ij} of $F(X,Y)$ for $p = 3$

$$a_{12} = -3m_1 = -v_1$$

$$a_{14} = 9m_1^2 = v_1^2$$

$$a_{16} = -27m_1^3 = -v_1^3$$

$$a_{18} = -9m_2 + 81m_1^4 = -3v_2$$

$$a_{23} = 27m_1^2 = 3v_1^2$$

$$a_{25} = -162m_1^3 = -6v_1^3$$

$$a_{27} = -36m_2 + 810m_1^4 = -12v_2 + 6v_1^4$$

$$a_{34} = -351m_1^3 = -13v_1^3$$

$$a_{36} = -84m_2 + 2943m_1^4 = -28v_2 + 27v_1^4$$

$$a_{45} = -126m_2 + 5346m_1^4 = -42v_2 + 52v_1^4$$

PART 2: The structure maps for $BP_*(BP)$

For $p = 2$

$$\eta_R(v_1) = v_1 + 2t_1$$

$$\eta_R(v_2) = v_2 + 2t_2 - 5v_1t_1^2 - 4t_1^3 - 3v_1^2t_1$$

$$\eta_R(v_3) = v_3 + 2t_3 - (v_2^2 + v_2v_1^3 + 2v_1^6)t_1 - (v_1^2v_2 + 11v_1^5)t_1^2 - (2v_2v_1 + 36v_1^4)t_1^3$$
$$- (2v_2v_1 + v_1^4)t_2 - (70v_1^3 - v_2)t_1^4 - (4v_2 + 2v_1^3)t_2t_1 - 85v_1^2t_1^5$$

$$- 2v_1^2t_2t_1^2 - 56v_1t_1^6 - 4v_1t_2t_1^3 - v_1t_2^2 - 16t_1^7 - 4t_1t_2^2$$

$$c(t_1) = - t_1$$

$$c(t_2) = - t_2 - v_1t_1^2 - t_1^3$$

$$c(t_3) = - t_3 - t_2^2t_1 - 3t_2t_1^4 - t_1^7 - v_1t_2^2 - 3v_1t_2t_1^3 - 3v_1t_1^6 - v_1^2t_2t_1^2 - 2v_1^2t_1^5$$
$$- v_1^3t_1^4 - v_2t_1^4$$

$$\psi(t_1) = t_1 \otimes 1 + 1 \otimes t_1$$

$$\psi(t_2) = t_2 \otimes 1 + 1 \otimes t_2 + t_1 \otimes t_1^2 - v_1t_1 \otimes t_1$$

$$\psi(t_3) = t_3 \otimes 1 + 1 \otimes t_3 + t_1 \otimes t_2^2 + t_2 \otimes t_1^4 - v_1t_2 \otimes t_2 - v_1t_1 \otimes t_2t_1^2$$
$$- v_1t_2t_1 \otimes t_1^2 + v_1^2t_1 \otimes t_2t_1 + v_1^2t_2t_1 \otimes t_1 + v_1^2t_1^2 \otimes t_1^3$$
$$- (2v_2 + v_1^3)t_1^3 \otimes t_1 - (2v_2 + v_1^3)t_1 \otimes t_1^3 - (3v_2 + 2v_1^3)t_1^2 \otimes t_1^2$$