$S$ is a compact, closed (i.e. $\partial S = \emptyset$) surface (not necessarily connected) Suppose $S$ is triangulated with $f$ triangles ($f$ is the number of faces). Then

$$2e = 3f$$

which implies

$$e = 3(v - \chi(S)).$$

Now there are at most $\binom{v}{2}$ edges (why?). So

$$v^2 - 7v + 6\chi(S) \geq 0.$$

This implies

$$v \geq 1/2(7 + \sqrt{49 - 24\chi(S)})$$

(fill in the details).

For example if $S = T^2$, $\chi(S) = 0$ and $v \geq 7 \Rightarrow f \geq 14$. There is a triangulation of the torus with exactly 14 triangles. I am sure it can be found on line.

If you have nothing to do and lots of time to do it in (apologies to Mae West) try to find all regular decompositions of the projective plane, $P$.(THIS IS NOT TO BE HANDED IN! ) Recall a decomposition is regular if each face has the same number of edges ($i$) and each vertex has the same number of edges ( $j$). We proved that there are 5 regular decompositions of the the sphere (called the Platonic solids).

Here is a bit of help. You will find that the case $i = 4, j = 3$ does not violate the Euler characteristic of $P$. Try to fit the squares on a surface. You will see that it is impossible. So this case and its dual do not exist. Another bit of a hint - Take a look at the dodecahedron. Does it respect the antipodal map?