SYLLABUS FOR MATH 83100. GROUP THEORY

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The course will be about various aspects of geometric, asymptotic, and algorithmic group theory and exciting connections between all the above. Geometric Group Theory is an actively developing area of mathematics. It is built on the ideas and techniques from low dimensional topology, Riemannian geometry, analysis, combinatorics, probability, logic and traditional group theory. One of the main ideas of Geometric Group Theory is to study the interaction between algebraic properties of a finitely generated group and geometric properties of a space admitting a nice isometric action of this group.

TOPICS:

1. Free groups, their properties and their subgroups via Stallings subgroup graphs. Residual finiteness and its generalizations.
2. Groups given by generators and relations. Cayley graphs and the word metric. Van Kampen diagrams and Van Kampen Theorem. Word-hyperbolic groups-finitely presented groups that exhibit a coarse form of negative curvature.
3. Groups actions on graphs by isometries and Bass-Serre theory, amalgamated free products and HNN extensions, graphs of groups and group actions on simplicial trees.
4. Group actions on Λ-trees, in particular, $\mathbb{R}$-trees.
5. Equations in groups and limit groups (=fully residually free groups).

Topics 4 and 5 correspond to methods and techniques used for the solution of the Tarski problems (Kharlampovich, Miasnikov, Sela) about first-order logic of a free group. This recent work opened a new direction of research called now "Algebraic geometry over groups". The theory developed over the years to solve the Tarski problems has uncovered deep connections between model theory, geometry and group theory. Another new direction of research is the theory of fully residually free groups (limit groups). Limit groups play a crucial role in the theory of equations and first-order formulas over a free group. It is remarkable that these groups, which have been widely studied before (after works by G. and B. Baumslags), turn out to be the basic objects in newly developing areas of algebraic geometry and model theory of free groups. These groups are exactly the coordinate groups of irreducible algebraic varieties over a free group; they have the same existential theory as a non-abelian free group, they can be obtained as limits of free groups in Gromov-Hausdorff-Grigorchuk topology. They are relatively hyperbolic and have many properties similar to those of free groups. The activity in this field has been growing incredibly fast, huge progress has been achieved, but a lot of work is yet to be carried out.

PREREQUISITES: some background in algebra (notion of a group, subgroup, quotient, homomorphism, action of a group on a set) or (and) topology (fundamental group, covering space)

The grades will be based on homework and projects.

Background reading There is no required textbook.

There will be notes on math.hunter.cuny.edu/olgak/teaching/grouptheory. Additional reading includes:

Pierre de la Harpe "Topics in Geometric group theory. Chicago lectures in Mathematics". University of Chicago Press, Chicago, IL, 2000.

Cominatorial Group Theory, by R. Lyndon and P. Schupp, Springer-Verlag, 2001; ("Classics in Mathematics series", reprint of the 1977 edition)

Metric Spaces of Non-positive Curvature, by M. Bridson and A. Haefliger, Springer, 1999

Groups Acting on Graphs, by W. Dicks and M. Dunwoody, Cambridge studies in advanced mathematics, vol. 17, Cambridge University Press, 1989

Introduction to group theory, by O. Bogopolski, 2008, EMS.

J. Stallings, Topology of finite graphs, Invent. Math. 71 (1983), no. 3, pp. 551-565

I. Kapovich and A. Myasnikov, Stallings foldings and subgroups of free groups, Journal of Algebra 248 (2002), pp. 608-668

O. Kharlampovich, A. Myasnikov, Equations and fully residually free groups, Combinatorial and geometric group theory, Trends in Mathematics, 203-242, Springer, 2010. http://arxiv.org/abs/0904.4482

M. Bestvina, M. Feighn, Stable actions of groups on real trees, Invent Math, 1995, v. 121, 2, p. 287-321. . M. Bestvina, $\mathbb{R}$-trees in topology, geometry and group theory, survey paper, available on his homepage.