

Syllabus for Combinatorial Group theory, Math 795.55.

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Geometric Group Theory is an actively developing area of mathematics. It is built on the ideas and techniques from low dimensional topology, Riemannian geometry, analysis, combinatorics, probability, logic and traditional group theory. One of the main ideas of Geometric Group Theory is to study the interaction between algebraic properties of a finitely generated group and geometric properties of a space admitting a nice isometric action of this group. The course will be about various aspects of geometric, asymptotic, and algorithmic group theory and connections between all the above.

TOPICS:

1. Free groups, their properties and their subgroups via Stallings subgroup graphs. Residual finiteness and its generalizations.
2. Groups given by generators and relations. Cayley graphs and the word metric. Van Kampen diagrams and Van Kampen Theorem. Word-hyperbolic groups-finitely presented groups that exhibit a coarse form of negative curvature.
3. Hyperbolic groups, quasi-isometries and quasiconvex subgroups.
4. Groups actions on sets. Groups actions on graphs by isometries and Bass-Serre theory, amalgamated free products and HNN extensions, graphs of groups and group actions on simplicial trees.

PREREQUISITES: some background in algebra (notion of a group, subgroup, quotient, homomorphism) or (and) topology (fundamental group, covering space)

The grades will be based on homework and projects.

Background reading There is no required textbook.

There will be notes on math.hunter.cuny.edu/olgak/teaching/grouptheory2012. Additional reading includes:

- paper by Scott and Wall, Topological methods in group theory.
- Pierre de la Harpe "Topics in Geometric group theory. Chicago lectures in Mathematics". University of Chicago Press, Chicago, IL, 2000.
- Combinatorial Group Theory, by R. Lyndon and P. Schupp, Springer-Verlag, 2001; ("Classics in Mathematics series", reprint of the 1977 edition)
- Metric Spaces of Non-positive Curvature, by M. Bridson and A. Haefliger, Springer, 1999
- Groups Acting on Graphs, by W. Dicks and M. Dunwoody, Cambridge studies in advanced mathematics, vol. 17, Cambridge University Press, 1989
- Introduction to group theory, by O. Bogopolski, 2008, EMS.
- J. Stallings, Topology of finite graphs, Invent. Math. 71 (1983), no. 3, pp. 551-565
- I. Kapovich and A. Myasnikov, Stallings foldings and subgroups of free groups, Journal of Algebra 248 (2002), pp. 608-668