Some problems on Markov Chains

1) For each of the following Markov chains, classify all the states as recurrent or transient. (Note: Definition 6.3.1 and Theorem 6.3.2 (b) are convenient here.)

   a) \[ P_1 = \begin{pmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix} \]
   
   b) \[ P_2 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \]
   
   c) \[ P_3 = \begin{pmatrix} 1/2 & 0 & 1/2 & 0 & 0 \\ 1/4 & 1/2 & 1/4 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 0 & 1/2 & 1/2 \end{pmatrix} \]
   
   d) \[ P_4 = \begin{pmatrix} 1/4 & 3/4 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1/3 & 2/3 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \]

The following two problems may be handed in for extra credit but it is not required.

2) Three out of every four trucks on the road are followed by a car, while only one out of every five cars is followed by a truck. What fraction of vehicles on the road are trucks? (Set this up as a Markov chain and compute the stationary distribution, whose components are the long run proportion of time spent in each state)

3) A certain town never has two sunny days in a row. Each day is classified as being either sunny, cloudy (but dry), or rainy. If it is sunny one day, then it is equally likely to be either cloudy or rainy the next day. If it is rainy or cloudy one day, then there is one chance in two that it will be the same the next day, and if it changes then it is equally likely to change to either of the other two possibilities. In the long run, what proportion of days are sunny? What proportion are cloudy?