1) Consider the Gamblers Ruin Problem (page 17, Example 1.7.4 for $p = 1/2$ and page 74, Example 3.9.6 for case of general $p$.) Let $D_k$ denote the average number of plays it takes for the gambler to either go broke or win, given that he starts with $k$ dollars. Compute $D_k$. In other words, for a random walk with probability $p$ of going up by one on each step, and probability $1 - p$ of going down by one on each step, with absorbing barriers 0 and $N$, what is the mean number of steps before hitting either of the absorbing barriers, starting at $k$. (Hint: verify equation (8), pg. 74, which is a second order, linear difference equation with constant coefficients. There is a link on our webpage to a succinct tutorial on how to solve such things.)

2) A prisoner is trapped in a cell with three doors. The first door leads to a tunnel that returns him to his cell after two days of travel. The second door leads to a tunnel that returns him to his cell after three days of travel. The third door leads immediately to freedom.

   a) Assuming that the prisoner will always select doors 1, 2, and 3 with probabilities 0.5, 0.3, and 0.2 respectively, what is the expected number of days until he reaches freedom?

   b) Assuming that the prisoner is always equally likely to choose among those doors that he has not yet used, what is the expected number of days until he reaches freedom? (So for example, if he first chooses door 1 and then returns to his cell, he will next choose, with equal likelihood, between doors 2 and 3.)

   c) For parts a) and b), find the variance of the number days until the prisoner reaches freedom.

3) Sometimes the notion of expected value, or average, doesn’t correspond to what we think of as the most likely thing to happen. Consider a stock which has a 50% chance of increasing by 80% by the end of a week, and a
50% chance of decreasing by 60% after a week. You start with $10,000 worth of the stock.

a) Compute the expected value (or average value) of your investment at the end of one year (52 weeks).

b) The above answer notwithstanding, what is the most likely value your investment will have?

c) What is the probability that your investment will be worth at least the average? (You’ll probably need some sort of calculator or spreadsheet to do this last part).