1. Let $X$ be a continuous random variable with a density function which is symmetric about 0. Show that if $E(X)$ is finite then it must be 0.

2. Let $X_1, X_2, \ldots, X_{100}$ be independent random variables with the common density $f(x) = 2 - 2x$, $0 \leq x \leq 1$. Let $S = X_1 + X_2 \ldots X_{100}$. Use the Central Limit Theorem to estimate $P(S \leq 35)$.

3. Compute the moment generating function of a geometric random variable, and use it to compute the mean and the variance.

4. In a certain community, 4 percent of all adults over the age of 50 have tuberculosis (T.B.). A health service in this community correctly diagnoses 98 percent of all persons with T.B. as having the disease, and incorrectly diagnoses 3 percent of all persons without T.B. as having the disease. Find the probabilities that,
   
   (a) the community health service will diagnose an adult over 50 as having T.B.,
   
   (b) a person over 50 diagnosed by the health service as having T.B. actually has the disease.

5. Suppose the joint probability density of $X$ and $Y$ is given by

   $$f(x, y) = \begin{cases} 
   24y(1 - x - y) & \text{for } x > 0, y > 0, x + y < 1, \\
   0 & \text{elsewhere.}
   \end{cases}$$

   (a) find the marginal density of $X$.
   
   (b) find the marginal density of $Y$.
   
   (c) determine if the two variables are independent.

6. Let $X$ have variance $\sigma^2$ and let $m_i = E(X^i)$ denote the $i$th moment. The skewness of the random variable $X$ is defined to be

   $$\text{skw}(X) = E((X - m_1)^3)/\sigma^3.$$ 

   (a) Show that

   $$\text{skw}(X) = \frac{m_3 - 3m_1m_2 + 2m_1^3}{\sigma^3},$$

   (b) Compute the skewness of an exponential variable with parameter $\lambda$ and show that it doesn’t depend on $\lambda$. (The best way to compute the moments is from the moment generating function.)
7. Suppose you play a series of 100 independent games. If you win a game, you win 4 dollars. If you lose a game, you lose 4 dollars. The chances of winning each game is 1/2. Use the central limit theorem to estimate the chances that you will win more than 50 dollars.

8. Let $X$ and $Y$ be continuous random variables, having joint probability density function

$$f(x, y) = \begin{cases} 24xy & \text{for } 0 < x < 1, \ 0 < y < 1, \ 0 < x + y < 1, \\ 0 & \text{elsewhere.} \end{cases}$$

Find the probability density of $Z = X + Y$.

9. (This is problem 14.19, pg. 116, from the book.) Compute the characteristic of a Gamma variable, with $\beta = 1$, i.e. with density $f(x) = \frac{1}{\Gamma(\alpha)} x^{\alpha - 1} e^{-x}$, $x \geq 0$ by the following method: use the Taylor series expansion of $e^{ix}$ and show that

$$E[e^{iuX}] = \frac{1}{\Gamma(\alpha)} \int_0^\infty e^{ix} x^{\alpha - 1} e^{-x} dx = \frac{1}{\Gamma(\alpha)} \sum_{n=1}^{\infty} \frac{(iu)^n}{n!} \int_0^\infty e^{-x} x^{n+\alpha - 1} dx = \sum_{n=0}^{\infty} \frac{\Gamma(n + \alpha)}{n!\Gamma(\alpha)} (iu)^n$$

and then show that is the binomial series for $\frac{1}{(1 - iu)^\alpha}$.

**NOTE:** The first sum starts at $n = 1$ and the second sum starts at $n = 0$. This is not a typo!