Consider the Initial Value Problem
\[ (0.1) \quad \frac{dy}{dx} = F(x, y) \quad y(x_0) = y_0 \]

**Theorem 0.2.** Suppose \( F(x, y) \) is continuous on an open rectangle in the plane which contains \((x_0, y_0)\) at the center, in other words, there are positive numbers \( \delta \) and \( \epsilon \) such \( F \) is continuous on
\[ R = \{(x, y)| x_0 - \delta < x < x_0 + \delta, y_0 - \epsilon < y < y_0 + \epsilon\}. \]
Then there exists a positive number \( \delta_1 \) such that there exists a solution \( f(x) \) to (0.1) defined on the interval \( x_0 - \delta_1 < x < x_0 + \delta_1 \). (Note: \( \delta_1 \) may be smaller than \( \delta \), i.e. the width of the interval on which the solution exists may be smaller than the width of the rectangle \( R \) in the hypothesis.)

**Theorem 0.3.** Suppose \( F(x, y) \) and \( \frac{\partial}{\partial y} F(x, y) \) are both continuous on a rectangle as in Theorem 0.2. Then there exists a positive number \( \delta_2 \leq \delta_1 \) such that the solution \( y = f(x) \) guaranteed by Theorem 0.2 is unique on the interval \( x_0 - \delta_2 < x < x_0 + \delta_2 \).

Do the following problems from the book, referring to the above theorems. In other words, for each of the Initial Value Problems listed, determine whether the IVP satisfies the hypothesis of Theorem 0.2, thereby guaranteeing a solution, and whether it satisfies Theorem 0.3, thereby guaranteeing the uniqueness of the solution. Note that you are not being asked to solve the differential equations - you’re just checking whether they satisfy the hypotheses of the theorems.

1.3/ 11, 13, 14, 17, 18.

Also, consider problem 14. If this IVP satisfies Theorem 0.2 but not Theorem 0.3, that would mean that it could have more than one solution on every small interval. Can you find two different solutions?