

TEACHING STATEMENT

VINCENT R. MARTINEZ

When I think of what it means to be a mathematician, I always end up asking myself what it means to be a teacher. Mathematicians are always communicating mathematics to others in many ways, through their publications, talks at conferences, the department tea-hour, as well as classroom lectures. At the heart of it then, we are teachers as much as we are explorers; in addition to pushing the frontiers of Mathematics further out through our work, we are constantly striving to impart clarity and understanding to it as well. There are many good ways to do this, but ultimately, I feel that one's own approach is hewn over time by practice and experience, as well as one's personality. What I hope to do here is give a brief portrait of my approach to teaching mathematics.

One of the things that I emphasize in any course is “intuition” for the concepts. Sometimes this intuition comes from our material experience with reality (Why does the “damping” in a differential equation decrease the energy?) or it can come from the underlying patterns built into the theory (Why do we hope for the operator we defined to be a self-map and a contraction?). On the other hand, I also find it helpful (and fun!) to develop an “artificial” intuition by metaphor or analogy, such as referring to arithmetical manipulations as “gymnastics” or a collection of equations as a “zoo of animals with different behaviors.” Regardless, I always try to associate an image to the concept whenever I can and if possible, draw a picture, e.g., by illustrating the method of characteristics as a loom that weaves out a surface. It is in this way mostly, that I try to make the subject concrete and imbue the concepts with a naturalness. That being said, intuition should be balanced by rigor, and so, without being overly pedantic, I try to emphasize what assumptions go into a given picture as well, for instance, or give warning to counterexamples that may be lurking beneath it.

A happy consequence I noticed in developing intuition this way is the confidence it seems to inspire in students to participate. Pictures can give abstract concepts a tangibility that students can criticize, and do! (Why do your domains always look like a kidney?) In every lecture, I always try to engage the students in a dialogue, even if it is rhetorical (What do we mean when we say, “This is the equation for the sphere?”). Only so much can be said in an hour lecture and even less effectively said if one loses the students' attention: every class is a battle to keep it and one of the victories that I strive for is to have them walk away from the lecture inspired to see the ideas through in the assigned problems.

Here is an outline of how I imagine a sample (somewhat theatric, admittedly), say, Vector Calculus class would go: 1) Start with a conspicuously penetrating question (What *is* the (x, y) -plane?), 2) Give familiar examples (rectangular coordinates), 3) Present problem that makes the question relevant (integrate a certain function over a circle), 4) Attempt solving the problem

using what is known at that point and practically fails (integrate with rectangular coordinates), 5) Introduce a new idea and solve the problem with it (polar coordinates), then finally, 6) Answer the original question (An artificial construction that depends on one's representation of a point; "The truth shall set you free!"), and 7) Develop the new idea further (cylindrical, spherical coordinates, etc.). However, certainly not all of my classes meet this ideal or go as planned, even if meticulous preparations are made. Indeed, some of my lectures have been directed by the students' questions.

When students make their own lines of inquiry that are relevant to the lecture's main ideas, I try to indulge them as best as I can, for it is in these moments that one can give the students a sense of ownership of their learning, as opposed to feeling that it is being delivered to them, and hopefully, inspire them to continue pursuing it. To keep the possibility of these moments open, I feel that it is important to allow some room for improvisation. Recently, for instance, I taught the notion of stability for initial value problems in my Introduction to Partial Differential Equations class and a student observed that it was reminiscent of the definition of a Lipschitz function. The student then asked if there was any relation to this and stability. I then smilingly segued into a light discussion on the notion of solution operators (which I introduced formally a few classes later) and concluded that indeed, the student's inkling was correct and specifically, that stability in this case could be seen as a consequence of these operators being Lipschitz continuous in a certain sense. To my pleasant surprise, several students in the class became curious and asked some rather interesting follow-up questions, such as what different norms the operator could be Lipschitz with respect to and what the solution operators for other equations were (we were discussing the heat equation at the time). I would like to believe that this impromptu episode was as memorable a moment for me as it was for my students.

Lastly, in assuming the role of the teacher now, I constantly remind myself of the influences that my own teachers had on me when I was a student. I would not be where I am today without the time, effort, and most importantly, the belief they collectively invested in me. To be a teacher is certainly more than just delivering good lectures; it is as well their duty to be a good mentor and advisor, to identify students with strong interest, encourage them, and develop their ability, which is something I quite take to heart.

To conclude, being a teacher is a fundamental part of the identity of a mathematician, and embedded in that role is also that of a mentor and advisor. As teachers, then, we are educators striving not only to communicate ideas in such a way that admits understanding, but to inspire and encourage the pursuit of understanding as well. On the other hand, we are also artists in the way we express ourselves through teaching. Indeed, while there are many effective approaches to teaching, in developing intuition through visualization and balancing it with rigor, encouraging an environment of active engagement, retaining an element of improvisation, while always being mindful of my roots as a student and taking example from my past teachers, I have found an approach that suits my own personality and gives me great joy in its practice. Because of this, in being a mathematician, I embrace the opportunity it gives me to also be a teacher.