TEACHING STATEMENT

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When I think of what it means to be a mathematician, I always end up asking myself what it means to be a teacher. Mathematicians are always communicating mathematics to others in many ways, through their publications, talks at conferences, department tea hour, and classroom lectures. At the heart of it then, we are teachers as much as we are explorers. In addition to pushing the frontiers of mathematics further out through our work, we are constantly striving to impart clarity and understanding to it as well. On the other hand, I do not believe that mathematics exists in a vacuum, without people. Indeed, it is a human endeavor and a very social one at that. I feel that it is important to keep this mind not only when doing mathematics, but also when communicating it. People create mathematics, guided by nature and each other, mathematicians and their students alike. What I hope to do presently is give a portrait of my philosophy to teaching mathematics.

One of the things that I emphasize in any course is "intuition" for the concepts. Sometimes this intuition comes from our material experience with reality (Why does the "damping" in a differential equation decrease the energy?) or it can come from the underlying patterns built into the theory (Why do we hope for the operator we defined to be a self-map and a contraction?). On the other hand, I also find it helpful (and fun!) to develop an "artificial" intuition by metaphor or analogy, such as referring to arithmetical manipulations as "gymnastics" or a collection of equations as a "zoo of animals with different behaviors." Regardless, I always try to associate an image to the concept whenever I can and if possible, draw a picture, e.g., by illustrating the method of characteristics as a loom that weaves out a surface. In doing so, I try to make the subject concrete and imbue the concepts with a sense of naturality. That being said, intuition should be balanced by rigor, and so, without being overly pedantic, I try to emphasize what assumptions go into a given picture as well or give warning to counterexamples that may be lurking beneath it.

A happy consequence I noticed in developing intuition this way is the confidence it seems to inspire in students to participate. Pictures can give abstract concepts a tangibility that students can criticize, and do! (Why do your domains always look like a kidney?) In every lecture, I always try to engage the students in a dialogue, even if it is rhetorical (What do we mean when we say, "This is the equation for the sphere?"). Only so much can be said in an hour lecture and even less effectively said if one loses the students' attention: every class is a battle to keep it and one of the victories that I strive for is to have them walk away from the lecture inspired to see the ideas through in the assigned problems.

Here is an outline of how I imagine a sample (somewhat theatric, admittedly), say, Vector Calculus class would go: 1) Start with a conspicuously penetrating question (What *is* the (x, y)-plane?), 2) Give familiar examples (rectangular coordinates), 3) Present problem that makes the question relevant (integrate a certain function over a circle), 4) Attempt solving the problem using what is known at that point and practically fails (integrate with rectangular coordinates), 5) Introduce a new idea and solve the problem with it (polar coordinates), then finally, 6) Answer the original question (An artificial construction that depends on one's conception of how a point in space is represented; "The truth shall set you free!"), and 7) Develop the new idea further (cylindrical, spherical coordinates, etc.). However, certainly not all of my classes meet this ideal or go as planned, even if meticulous preparations are made. Indeed, some of my lectures have been directed by the students' questions.

When students make their own lines of inquiry that are relevant to the lecture's main ideas, I try to indulge them as best as I can, for it is in these moments that one can give the students a sense of ownership of their learning, as opposed to feeling that it is being delivered to them, and hopefully, inspire them to continue pursuing it. To keep the possibility of these moments open, I feel that it is important to allow some room for improvisation. On the other hand, students can often ask a variety of questions, some of which belie a deep insight, while others have a performative quality to it and serve to show others or perhaps themselves that they know more than what is being taught. Some of us mathematicians are guilty of some forms of this as well. However, such behavior serves not only to reinforce our apparently brittle egos, but more importantly, the idea that mathematics is an esoteric subject that only those "pre-ordained" with the ability to penetrate its secrets have the privilege to practice. This cannot be further from the truth. Talent can come from anywhere, from anyone. It is thus fundamentally important to communicate mathematics in ways that promote inclusivity and dispel the notion that only a select few have access to it. One of the ways I do this in the classroom is to interpret student inquiries or comments in a positive light that shows the value of their way of thinking, if even if it is to demonstrate how it leads to other productive ways of looking at the problem at hand. In the end, I concede that not everyone sees mathematics in the ways I do, but should in no way preclude others from experiencing the joy that comes its learning and doing. I therefore assume full responsibility for facilitating this experience to any and all who are in my classroom.

Lastly, having been a teacher for some time now, I am constantly reminded of the influences that my own teachers have had on me as a student. I would not be where I am today without the time, effort, and belief they collectively invested in me. To be a teacher is certainly more than just delivering good lectures. It is as well their duty to be a good mentor, to identify students with strong interest, encourage them, and develop their ability. It is something I take to heart. Having mentored several students at both the undergraduate and graduate level, I've seen how incredible a student's development can be in so little time, from struggling to writing proofs to writing beautiful theses imbued with their own ideas. Two important lessons I learned from these experiences are: 1) when students are struggling with the material, one can always adjust the level of the content without compromising rigor, and 2) to not be afraid to invest time into working with them. The first lesson has allowed me to practice my craft honestly without betraying either mathematics or my students, while the second lesson has allowed me to more deeply engage with both. Indeed, I've seen that when a mutual trust is established between student and mentor, there is a great capacity for inspiration from each other, which can then open the door to learning and doing mathematics strongly and together.

To conclude, being a teacher is a fundamental part of the identity of a mathematician, and embedded in that role is also that of a mentor and advisor. As teachers, then, I believe we should strive not only to communicate ideas in such a way that admits understanding, but to inspire and encourage the pursuit of understanding as well in an inclusive a manner as possible. On the other hand, we are also artists in the way we express ourselves through teaching. Indeed, while there are many effective approaches to teaching, in developing intuition through visualization and balancing it with rigor, encouraging an environment of active engagement, retaining an element of improvisation, while always being mindful of my roots as a student and taking example from my past teachers, I have found an approach that suits my own personality and gives me great joy in its practice. Because of this, in being a mathematician, I embrace the opportunity it gives me to also be a teacher.