Basic Probability Summer 2009
NYU Courant Institute
Midterm Exam

1. Suppose that an airplane engine will fail, when in flight, with probability $1-p$ independently from engine to engine; suppose that the airplane will make a successful flight if at least 50% of its engines remain operational. If $p = 3/4$, which is preferable, a four-engine plane or a two-engine plane? What about if $p = 1/2$?

2. Each customer that enters 'Reasonably Honest Dave's Appliance Store' will purchase a TV with probability p. If the number of customers entering the store is Poisson distributed with mean λ, what is the probability that Dave will sell k TVs? If X is the number of TVs sold, what kind of distribution does X have, and what is the mean of X? (Hint: compute $P(X = k)$ by conditioning on N, the number of customers entering the store.)

3. A TV purchased from 'Reasonably Honest Dave's Appliance Store' will require repair on the average once every two years. Assuming that the times between repairs are exponentially distributed, what is the probability that the TV will work at least 3 years without requiring repairs?

4. Let X and Y be independent Bernoulli variables with parameter $p = 1/2$. Show that $X+Y$ and $|X-Y|$ are uncorrelated but not independent.

5. Compute the mean and variance of a random variable with a gamma distribution with parameters t and λ. (Recall that the density function for the gamma distribution is given by $f(x) = \dfrac{\lambda^t}{\Gamma(t)} x^{t-1} e^{-\lambda x}, \quad x \geq 0.$)

6. Let U be a uniform random variable on $[0, 1]$, and let $V = \dfrac{1}{U}$.

a) Find the density function of V. (HINT: first compute the cdf of V, then differentiate.)

b) What is the mean of V?