

Basic Probability Summer 2009
NYU Courant Institute
Practice Problems

1. Which is more likely: 9 heads in 10 tosses of a fair coin or 18 heads in 20 tosses?
2. Let X be a binomially distributed random variable with n trials and probability p of success on each trial. For which value of k is $P(X = k)$ maximized? (Hint: Consider $\frac{P(X=k)}{P(X=k-1)}$, the ratio of successive terms. If this ratio is bigger than one, then the probability is still going up with k , if it is smaller than one, then it is going down.)
3. Three identical fair coins are thrown simultaneously until all three show the same face. What is the probability that they are thrown more than three times?
4. Let X and Y be jointly continuous random variables with joint density

$$f(x, y) = \begin{cases} \frac{6}{7}(x + y)^2 & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

- a) Find (i) $P(X > Y)$, (ii) $P(X + Y \leq 1)$.
 - b) Find the marginal densities of X and Y .
 - c) Are X and Y independent? Why or why not?
5. When taping a television commercial, the probability is 0.30 that a certain actor will get his lines straight on any one take. What is the probability that he will get his lines straight for the first time on the sixth take?
 6. Suppose that in a certain city the number of muggings can be approximated by a Poisson process with $\lambda = 4$ per month.
 - a) Find the probability of there being 48 muggings in a year.
 - b) What is the probability of 3 muggings in one week?
 7. The lifetime of a transistor radio is T years, where T is an exponential variable with parameter $\lambda = .5$. What is the probability that the radio will last at least 5 years? Given that it lasts 10 years, what is the probability that it will last at least 5 more years beyond that?
 8. Let X be uniform on $[0, 1]$.
 - a) Find the density function of X^3 and use this to compute $E(X^3)$.
 - b) Use the Law of the Unconscious Statistician to find $E(X^3)$.
 9. It is raining cats and dogs. The number of animals N that fall in a given area during a given interval of time is Poisson with parameter λ . Each animal that falls has probability p_c of being a cat, and probability p_d of being a dog, where $p_c + p_d = 1$. Find the average number of cats falling in the given area during the given interval of time.
 10. Let X be gamma distributed with parameters t and λ . Compute $E(1/X)$ for those values of t and λ for which it exists.
 11. Show that $\Gamma(1/2) = \sqrt{\pi}$.