Basic Probability Summer 2009
NYU Courant Institute
Practice Problems for the Final Exam

1. Let X be a continuous random variable with a density function which is symmetric about 0. Show that $E(X) = 0$.

2. Each morning a person leaves her house and goes for a run. She is equally likely to leave either from the front door or the back door. Upon leaving she chooses a pair of running shoes from among those sitting by the door, or goes running barefoot if there are no shoes there. On her return, she is equally likely to enter by the front door or the back, and she removes her shoes, if she is wearing any , and leaves them by the door. If she owns a total of n pairs of running shoes, what proportion of the time does she run barefoot? (Hint: set this up as a Markov chain and compute the stationary distribution.)

3. Let $X_1, X_2, \ldots, X_{100}$ be independent random variables with the common density $f(x) = 2-2x$, $0 \leq x \leq 1$. Let $S = X_1 + X_2 \ldots X_{100}$. Use the Central Limit Theorem to estimate $P(S \leq 35)$.

4. Compute the moment generating function of a geometric random variable, and use it to compute the mean and the variance.

5. In a certain community, 4 percent of all adults over the age of 50 have tuberculosis (T.B.). A health service in this community correctly diagnoses 98 percent of all persons with T.B. as having the disease, and incorrectly diagnoses 3 percent of all persons without T.B. as having the disease. Find the probabilities that,

a) the community health service will diagnose an adult over 50 as having T.B.,

b) a person over 50 diagnosed by the health service as having T.B. actually has the disease.

6. Suppose the joint probability density of X and Y is given by

$$f(x,y) = \begin{cases} 24y(1-x-y) & \text{for } x>0,\ y>0,\ x+y<1, \\ 0 & \text{elsewhere.} \end{cases}$$

a) find the marginal density of X.

b) find the marginal density of Y.

c) determine if the two variables are independent.

7. Let X have variance σ^2 and let $m_i = E(X^i)$ denote the ith moment. The *skewness* of the random variable X is defined to be

$$\text{skw}(X) = E((X - m_1)^3)/\sigma^3.$$

a) Show that

$$\text{skw}(X) = \frac{m_3 - 3m_1m_2 + 2m_1^3}{\sigma^3},$$

b) Compute the skewness of an exponential variable with parameter λ and show that it doesn't depend on λ. (The best way to compute the moments is from the moment generating function.)

8. Suppose X is a Poisson variable, with parameter λ. For which value of k is $P(X = k)$ the greatest? Hint: compare successive values, i.e. look at

$$\frac{P(X = k)}{P(X = k - 1)}.$$

9. Suppose you play a series of 100 independent games. If you win a game, you win 4 dollars. If you lose a game, you lose 4 dollars. The chances of winning each game is 1/2. Use the central limit theorem to estimate the chances that you will win more than 50 dollars.

10. Let X and Y be continuous random variables, having joint probability density function

$$f(x, y) = \begin{cases} 24xy & \text{for } 0 < x < 1,\ 0 < y < 1,\ 0 < x + y < 1, \\ 0 & \text{elsewhere.} \end{cases}$$

Find the probability density of $Z = X + Y$.