Basic Probability Summer 2009
NYU Courant Institute
Quiz- Solutions

1. A gambler has in her pocket a fair coin and a two-headed coin.

a) She selects one of the coins at random, and when she flips it, it shows heads. What is the probability that it is the fair coin?

Solution: Let F = 'the coin is fair', H = 'heads on first toss', HH = 'heads on first and second toss'. Then $P(F|H) = \frac{P(F \cap H)}{P(H)} = \frac{P(H|F)P(F)}{P(H|F)P(F) + P(H|F^c)P(F^c)} = \frac{(1/2)(1/2)}{(1/2)(1/2) + (1) \cdot (1/2)} = \frac{1}{3}$.

b) Suppose that she flips the same coin a second time and again it shows heads? Now what is the probability that it is the fair coin?

Solution: $P(F|HH) = \frac{P(F \cap HH)}{P(HH)} = \frac{P(HH|F)P(F)}{P(HH|F)P(F) + P(HH|F^c)P(F^c)} = \frac{(1/4)(1/2)}{(1/4)(1/2) + (1) \cdot (1/2)}$ 1/5.

c) Suppose that she flips the same coin a third time and it shows tails? Now what is the probability that it is the fair coin?

Solution: The answer is obviously 1, since if the coin show tails at all, it can't be the two headed coin.

2. A mouse has two levers, and is given food if it presses the left hand lever, an electric shock if it presses the right hand lever. The first time there is a 50-50 chance it will press either lever. If it presses the left lever and gets food the first time, then there is a probability of .70 that it will press the left lever the second time. If it presses the right lever and gets a shock the first time, then there is a probability of .80 it will press the left lever the second time.

a) What is the probability the mouse will get food the second time?

Solution: Using an obvious notation:

$$P(F_2) = P(F_2|F_1)P(F_1) + P(F_2|S_1)P(S_1) = (.70)(.5) + (.80)(.5) = 0.35 + 0.4 = .75.$$

b) What is the probability that it got food the first time, given that it gets food the second time? (Hint: you can use Baye's Theorem, or else you can just draw a tree diagram.)

Solution:

$$P(F_1|F_2) = \frac{P(F_2 \cap F_1)}{P(F_2)} = \frac{P(F_2|F_1)P(F_1)}{P(F_2)} = \frac{.35}{.75} = .4667.$$

3. An elevator containing five people can stop at any of seven floors. What is the probability that no two people get off at the same floor? Assume that the occupants act independently and that all floors are equally likely for each occupant.

Solution: The sample space Ω consists of all the possible ways that the five people can get off at the seven floors. There are seven possibilities for the first person, seven for the second person, etc. so we have $|\Omega| = 7^5$. Let A be the event 'no two get off at the same floor'. Then $|A| = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3$. We get $P(A) = \frac{|A|}{|\Omega|} = .1499$.

4. Suppose we toss two fair dice.

a) Let E_1 be the event that the sum is six, and let F be the event that the first die is four. Are E_1 and F independent? Justify your answer with both a calculation and an intuitive argument.

Solution: We have $P(E_1) = \frac{5}{36}$, $P(F) = \frac{1}{6}$, so $P(E_1)P(F) = \frac{5}{216}$. However $P(E_1 \cap F) = \frac{1}{36}$ which is not the same, so E_1 and F are dependent.

b) Now let E_2 be the event that the sum of the dice is seven. Is E_2 independent of F? Again, justify your answer with a calculation and an intuitive argument.

Solution: We have $P(E_2) = \frac{1}{6}$, $P(F) = \frac{1}{6}$, so $P(E_2)P(F) = \frac{1}{36} = P(E_2 \cap F)$ so E_2 and F are independent.