1. (a) Show that $X$ is regular if and only if given any point $x \in X$ and any open set $U$ containing $x$, there is an open set $V$ such that $x \in V \subset U$.

(b) Show that if $\{X_\alpha\}$ is a family of spaces, and $A_\alpha \subset X_\alpha$ for each $\alpha$, then $\prod_\alpha A_\alpha = \prod_\alpha \overline{A_\alpha}$.

(c) Show that if each $X_\alpha$ is regular then the product $\prod_\alpha X_\alpha$ is regular.

2. Prove that if $X$ has only finitely many components, then all the components are open.

3. Prove that the product of finitely many compact spaces is compact without using the Tychonoff Theorem or anything equivalent to the axiom of choice.