1. Draw pictures for the maps in the Mayer Vietoris theorem. Use your pictures to “intuitively explain” the exactness.

2. (*) Prove a relative version of the Mayer Vietoris theorem:
   Suppose \( X \subset Y \) \( X = A \cup B \) satisfying the excision conditions, then there is an exact sequence
   \[
   \rightarrow H_n(Y, A \cap B) \rightarrow H_n(Y, A) \oplus H_n(Y, B) \rightarrow H_n(Y, X) \rightarrow H_{n-1}(Y, A \cap B) \rightarrow
   \]

3. (*) Show that if
   \[
   0 \rightarrow A \rightarrow B \rightarrow \mathbb{Z}^k \rightarrow 0
   \]
   is exact then
   \[
   B = A \oplus \mathbb{Z}^k.
   \]

4. (*) Use the Mayer Vietoris sequence to compute the homology of the Klein bottle.