MATH 260 HW 2

Hand In

(1) Let $W_1$ and $W_2$ be subspaces of a vector space $V$. Prove that $W_1 \cup W_2$ is a subspace of $V$ if and only if $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$.

(2) Let $W_1$ and $W_2$ be subspaces of a vector space $V$.
   a) Prove that $W_1 + W_2$ is a subspace of $V$ that contains both $W_1$ and $W_2$.  
   
   (Note: You must prove first that $W_1 + W_2$ is a subspace of $V$ and then prove that it contains $W_1$ and $W_2$.)

   b) Prove that any subspace of $V$ that contains both $W_1$ and $W_2$ must also contain $W_1 + W_2$.

   Note: If $S_1$ and $S_2$ are nonempty subsets of a vector space $V$, then the sum of $S_1$ and $S_2$, denoted $S_1 + S_2$, is the set \{ $x + y : x \in S_1$ and $y \in S_2$ \}.

(3) Suppose $b \in \mathbb{R}$. Prove that the set of all continuous real-valued functions on $[0,1]$ such that $\int_0^1 f = b$ is a subspace of $\mathcal{F}([0,1], \mathbb{R})$ if and only if $b = 0$.

(4) Let $V$ be a vector space over $F$ and let $u, v \in V$ be distinct. Prove that $\{u, v\}$ is linearly dependent if and only if one is a multiple of the other.

(5) Suppose $\{v_1, \ldots, v_n\}$ is linearly independent in a vector space $V$ and $w \in V$. Prove that if $\{v_1 + w, \ldots, v_n + w\}$ is linearly dependent, then $w \in \text{span}(\{v_1, \ldots, v_n\})$.

DNHI

Friedberg p. 20 #8,  
p. 33 #5, #13