1. Let $X$ be a continuous random variable with a density function which is symmetric about 0. Show that $E(X) = 0$.

3. Let $X_1, X_2, \ldots, X_{100}$ be independent random variables with the common density $f(x) = 2-2x$, $0 \leq x \leq 1$. Let $S = X_1 + X_2 \ldots X_{100}$. Use the Central Limit Theorem to estimate $P(S \leq 35)$.

4. Compute the moment generating function of a geometric random variable, and use it to compute the mean and the variance.

5. In a certain community, 4 percent of all adults over the age of 50 have tuberculosis (T.B.). A health service in this community correctly diagnoses 98 percent of all persons with T.B. as having the disease, and incorrectly diagnoses 3 percent of all persons without T.B. as having the disease. Find the probabilities that,

   a) the community health service will diagnose an adult over 50 as having T.B.,

   b) a person over 50 diagnosed by the health service as having T.B. actually has the disease.

6. Suppose the joint probability density of $X$ and $Y$ is given by

   $$f(x, y) = \begin{cases} 
   24y(1-x-y) & \text{for } x > 0, y > 0, x+y < 1, \\
   0 & \text{elsewhere.}
   \end{cases}$$

   a) find the marginal density of $X$.

   b) find the marginal density of $Y$.

   c) determine if the two variables are independent.
7. Let $X$ have variance $\sigma^2$ and let $m_i = E(X^i)$ denote the $i$th moment. The skewness of the random variable $X$ is defined to be

$$\text{skw}(X) = E((X - m_1)^3)/\sigma^3.$$ 

a) Show that

$$\text{skw}(X) = \frac{m_3 - 3m_1m_2 + 2m_1^3}{\sigma^3},$$ 

b) Compute the skewness of an exponential variable with parameter $\lambda$ and show that it doesn’t depend on $\lambda$. (The best way to compute the moments is from the moment generating function.)

9. Suppose you play a series of 100 independent games. If you win a game, you win 4 dollars. If you lose a game, you lose 4 dollars. The chances of winning each game is 1/2. Use the central limit theorem to estimate the chances that you will win more than 50 dollars.

10. Let $X$ and $Y$ be continuous random variables, having joint probability density function

$$f(x, y) = \begin{cases} 
24xy & \text{for } 0 < x < 1, 0 < y < 1, 0 < x + y < 1, \\
0 & \text{elsewhere}.
\end{cases}$$

Find the probability density of $Z = X + Y$. 